EARTH PRESSURE ON RETAINING WALLS

BY

MORRIS FAINSTEIN

ARMOUR INSTITUTE OF TECHNOLOGY

1920

721.1
F 14

Illinois Institute
of Technology
UNIVERSITY LIBRARIES

AT 534
Fainstein, M.
Method of measuring earth
pressures on vertical and

For Use In Library Only

Method of Measuring Earth Pressures on Vertical and Inclined Retaining Walls

---

A THESIS

PRESENTED BY

Morris Fainstein

TO THE

PRESIDENT AND FACULTY
OF
ARMOUR INSTITUTE OF TECHNOLOGY

FOR THE DEGREE OF

Bachelor of Science
IN
CIVIL ENGINEERING

May 1920

APPROVED:

Alfred E. Phillips
PROFESSOR OF CIVIL ENGINEERING

H. M. Raymond
DEAN OF ENGINEERING STUDIES

DEAN OF CULTURAL STUDIES

ILLINOIS INSTITUTE OF TECHNOLOGY
PAUL V. GALVIN LIBRARY
35 WEST 33RD STREET
CHICAGO, IL 60616

Digitized by the Internet Archive
in 2009 with funding from
CARLI: Consortium of Academic and Research Libraries in Illinois

http://www.archive.org/details/methodofmeasurin00fain

T 5[illegible]
F17

I take this opportunity of expressing my appreciation to Professor Alfred E. Phillips and Professor John C. Penn for the many valuable suggestions given during the preparation of this thesis.

25103

TABLE OF CONTENTS

PART ONE Page.

Theories of Earth Pressures on Retaining Walls.

Rankine's Theory 1

Coulomb's Theory 4

Weyrauch's Theory 5

Trautwine's Solution 6

PART TWO

Experiments.

M.M.L.Leygue's Experiments 8

Baker's Experiments 9

Lieutenents Hope's Experiments ..... 10 - 11

General Pasley's Experiments 12

W. Cain's Experiments ..... 13 - 14

E.P.Goodrich's Experiments ..... 15 - 16

PART THREE

Section 1 (Method) ..... 17 - 20

Section 2 (The Inclined Wall) ..... 21 - 26

Section 3 (The Vertical Wall) ..... 27 - 28

Section 4 (Design of Wall) ..... 29 - 33

Section 5 (Computations) ..... 34 - 35

I N T R O D U C T I O N

Retaining walls belong to that type of structures which is often dealt with by the engineer, but at the same time he knows very little of the actual forces which they will have to resist after completion. This uncertainty is due to lack of experimental data, and the designer must use mathematical conclusions as the only available guide in carrying out his design.

It is intended, therefore, to give in this thesis a brief account of the theories of earth pressures, to review the previous laboratory and field experiments, and to devise a method of measuring earth pressures on inclined and vertical retaining walls.

P A R T O N E

Theories.

Rankine's Theory.

The most prominent discussion between the various theories on the existing conditions of stress within a granular mass such as earth, sand, gravel or broken stone, which all, singularly or in combination, may constitute the fill in back of a retaining wall, is known as Rankine's Theory or Rankine's Ellipse of Stress, which was introduced by Professor Rankine in 1858.

This theory assumes that the above-mentioned materials consist of an incompressible, homogeneous, granular mass, the particles of which are held together by friction, the cohesive forces of the grains are neglected,[1] and are subjected to their own weight.

Note:

Later it has been proven experimentally that the cohesive force is negligible in these materials.

The most important conclusion arrived at by Professor Rankine is that if a mass of earth is under the action of its own weight, then any elementary prism as "A" within that mass is held in place by the vertical pressure "p" from above which produces an equal pressure from below, and a horizontal pressure "q" is caused by the pressure "p". The relation of the two as derived at by Professor Rankine is, *Fig. 1*

$$\frac{q}{p} = \frac{1 - \sin \theta}{1 + \sin \theta}$$

where "p" is the weight of the material and "q" is the angle of repose of the material. Now if we introduce a plane at any point of this granular mass the normal of which makes an angle θ with the vertical, the resultant pressure on the plane may be computed by the equations,

$r_t = (p - q) \sin \theta . \cos \theta$ = tangential component of resultant.

$r_n = p \cos^2\theta + q \sin^2\theta$ = normal component of resultant. *Fig. 2.*

Fig. 1.

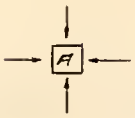
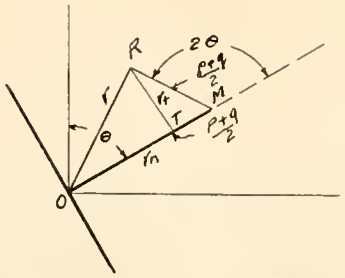


Fig. 2.



This is the second important conclusion to which the theory leads. If this plane is revolved about, "o","p" and "q" will remain constant,and the only variables are θ and "r",the latter changing with θ in direction and magnitude,and the locus of "r" is an ellipse with "p" and "q" as the major and minor axes. This is the last proof Professor Rankine offers.

According to this theory the pressure on a retaining wall is caused by a sliding prism of earth extending from the back of the wall to the plane of repose of the material. As shown in Fig. 3. below,A B is the back of the wall,θ is the angle of repose,and plane B C is the plane of repose. The pressure produced by this sliding on the wall is to be computed by the ellipse of stress.

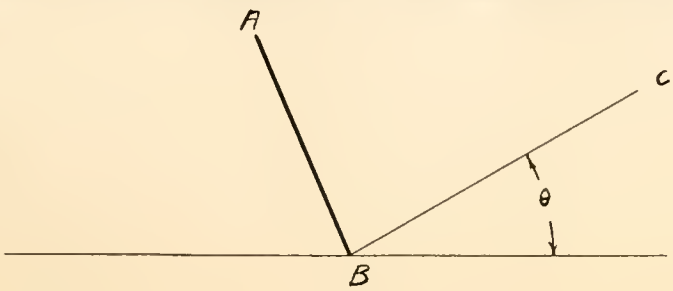


Fig. 3.

Coulomb's Theory.

Mr. Coulomb, in addition to the assumptions made by Rankine, introduces the "Plane of Rupture". According to this theory the pressure on the plane of the wall is due to a sliding prism of material, which has for its limits the wall on one side, and a plane which lies between the plane of repose and the plane of the wall, called the "Plane of Rupture".

This theory leads to two equations for computing the pressure,

$$P = \tfrac{1}{2}\, w\, h\, k$$

where P is the total pressure on the wall, w is the weight of a cubic unit of fill, and h is the height of the wall.

$$K = \frac{\operatorname{Sin}^2 (\theta - \phi)}{\operatorname{Sin}^2\theta . \operatorname{Sin} (\theta + \phi') \left(1 + \sqrt{\frac{\operatorname{Sin} (\phi + Z) \operatorname{Sin} (\phi - \alpha)}{\operatorname{Sin} (\theta + Z) \operatorname{Sin} (\theta - \alpha)}}\right)^2}$$

where Z is the angle that P makes with the normal to the wall, α is the angle of surcharge, and ϕ' is

the angle of friction of the filling on the wall. In order to compute P, the value of the angle Z has to be assumed; Z may be given any value between 0 and ϕ' or 0 and ϕ , in either case it is an elaborate assumption.

This theory known as the Sliding Wedge Theory was originated in 1687 by a French army General, it was further improved upon in 1729 by Vauban, and in 1773 Captain Coulomb gave it its present form.

Weyrauch's Theory.

Dr. J. J. Weyrauch in addition to the assumption made by Rankine assumes that the forces upon any plane section thru a mass of earth have the same direction. After a long discussion he obtained the two following formulas - one for the magnitude of the resultant pressure, and the other for its direction.

$$P = \frac{\cos(\phi + \alpha)}{(n+1)\cos\alpha} \times \frac{h^2 w}{2\cos(\alpha+\delta)}$$

δ is the angle which P makes with the normal to the wall, and α is the angle between the plane

of the wall and the vertical.

$$n = \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \varepsilon)}{\cos(\alpha + \delta)\cos(\alpha - \varepsilon)}}$$

ε is the angle of surcharge.

$$\tan(\alpha + \delta) = \frac{2\sin\varepsilon}{k\cos\phi} + \frac{\tan(\alpha - \varepsilon)}{k^2}$$

$$k = \frac{\cos\sqrt{\cos^2 - \cos^2\phi}}{\cos\phi}$$

Trautwine's Solution.

Mr. Trautwine assumes that the plane of rupture for vertical walls is half way between the back of the wall and the plane of repose, and he comes to the same results as does Coulomb.

Cain's Solution.

Professor Cain assumes that the resultant thrust makes an angle with the normal to the wall equal to the angle of friction between the filling and the wall (ϕ'). Professor Cain's solution gives the same results as does Coulomb's Theory.

There are many more solutions graphical and analytical having as their basis Coulomb's Theory.

P A R T T W O

Experiments.

The following is an account of the most important experiments, which were performed by various methods and observers on retaining walls, in order to determine the lateral pressure on the back of a wall and the point of application of the resultant.

M.M.L.Leygue's Experiments.

Mr. Leygue performed his experiment with a small model retaining wall using dry sand for backing. He gives the following as conclusions derived from the data obtained.

(1) That there are cohesive forces between the particles of sand, but they are so small that they may be neglected.

(2) That the surface of rupture is convex, and roughly bisects the angle between the back of the wall and the plane of repose.

(3) That the ratio of the horizontal to the vertical pressure for vertical walls with sand in horizontal layers is 0.3, and for the same walls having a surgare sloping at an angle equal to the angle of repose is 0.65

(4) The point of application of the resultant is from 0.296 h to 0.375 h, which gives an average of 0.34 h .

Mr. Baker's Experiments.

A wall 4 feet high and 1 foot thick of pitch pine blocks sustained the pressure of broken stone piled to a height of 3 feet 9 inches and 5 feet width on the top.

Calculations show that according to the theories of pressure the thickness of the wall in order to balance the thrust caused by this mass of stone had to be 1.01 feet thick, if the specific gravity of the wall was equal to that of the backing; but in the given case the wall was built of pine blocks which are much lighter than the stone, and had a thickness of 1 foot and resisted the thrust, since the existing theories do not give the actual conditions of pressures on retaining walls.

Lieutenant Hope's Experiments.

Experiment 1.

...

Lieutenant Hope measured the lateral pressure of sand on a board 1 foot square. The mean result of several experiments was 9 lbs. 7 oz., which corresponds to a pressure of fluid weighing 19 lbs. per cubic foot. The slope of the sand was 1.42 to 1, and the theoretical fluid pressure, therefore, is 26.2 lbs., which is about 40 per cent greater than the observed.

Experiments with gravel gave the same divergence from the theory.

Experiment 2.

...

To determine the position of the plane of rupture, Lieutenant Hope experimented upon colored sand placed in a glass vessel. It was found that the plane of rupture made an angle of 24 degrees with the vertical instead of 28 degrees as Mr. Coulomb's theory would give for these conditions.

Observations on a gravel bank indicated the same defect of the theory.

Experiment 3.

...

Lieutenant Hope built retaining walls of bricks laid in wet sand. The first of these walls had a vertical back, it was 20 feet long and 1 foot 11 inches thick; when the wall was raised to a height of 8 feet it was backed with ballast and under its pressure inclined 1 ½ inches from the vertical; at a height of 9 feet there was an inclination of 3 ¼ inches; and at a height of 10 feet it failed, at this instance the overhang was 4 inches.

Assuming that the pressure per linear foot was not more than 2000 lbs., and reducing it to fluid pressure, Lieutenant Hope obtained for the lateral thrust which caused the wall to fail equal to that of a fluid weighing 10.8 lbs. per cubic foot, instead of 23.6 lbs. which the theory indicates.

This shows that a wall being theoretically on the point of failure actually has a factor of safety of 2 .

The same was found to be true with similar walls built by Leutenent Hope.

General Pasley's Experiments.

General Pasley experimented upon a model retaining wall 3 feet 26 inches high; the pressure was measured by pulling over the wall by weights. From several hundred of such experiments the following conclusions were derived:

(1) The center of pressure is at one third of the height of the wall measured from the base.

(2) The pressure on the wall was equivalent to that of a fluid weighing 8.5 lbs. per cubic foot, rather than 21 lbs. per cubic foot as predicted by theories for the given conditions.

Mr. W. Cain's Experiments.

In the discussion of Lieutenant Hope's and Mr. Baker's experiments, Mr. Cain proves that if the friction of the filling on the back of the wall is taken into consideration, the results of the described experiments are in accordance with Coulomb's Theory.

Experiment 1.

...

Two boards hinged at the top were employed by Mr. Cain. One of the boards was backed by sand, and the other served as a support.

By this experiment it was found that Mr. Rankine's theory was applicable only when the retaining board was placed at or below the plane of rupture.

Experiment 2.

...

A small model retaining wall 2.952 feet high and 0.0755 feet thick made of brick and Portland cement was backed by sand with a level surcharge of 4.26 feet high above the wall. The pressure was measured at the limit of stability.

The pressure was measured at the limit of stability.

The experiment gave for the angle of friction of the filling on the back of the wall as 35^{o}.

From the same experiment the value of "C", which is the ratio of the distance from the foot of the wall to the point of application of the resultant pressure, to the height, was determined for various values of h'/h , where h' is the height of surcharge, and h is the height of the wall.

h'/h	C	h'/h	C	h'/h	C
	.333	1.5	.356	1.00	.364
2.00	.353	1.25	.360	.75	.364
		1.11	.362	.50	.364

Mr. E.P.Goodrich's Experiments.

Experiment 1.

...

A series of experiments were conducted by Mr. Goodrich upon a model which consisted of a box 3 feet square and 6 feet deep, the walls of the box were so constructed that they could easily move inside under an external force, the pressure caused by sand which filled the box was measured by the aid of levers and weights. It was found that the lateral pressure was approximately equal to 1/15 of the vertical pressure plus 15 lbs.

Experiment 2.

...

A box 6 x 1 x 15 feet (inside dimensions) was formed by driving 3 x 10 inch x 15 feet planks of yellow pine 1 foot from the face of a firm wall; the planks were backed with beach sand, and the deflections of the planks at various points were measured.

In order to compute the pressure on the wall from the deflections, similar planks were

supported at points 10 feet apart and loaded with bricks until the deflections caused were equal to those measured on the wall.

It was found that a pressure of 600 lbs. per square foot at a depth of 10 feet was exerted on the wall.

Mr. Goodrich concludes that this figure is too high as the fine beach sand could not give a pressure greater than 500 lbs. per square foot at this depth.

P A R T T H R E E

Section 1.

Method.

In the experiments described no satisfactory data has been obtained which would enable us to check the various theories of earth pressures on retaining walls; the following method will furnish a means by which both the resultant pressure and its direction may be determined experimentally.

This method consists of measuring separately the vertical and horizontal components of the resultant thrust on the back of the wall by the following arrangement.

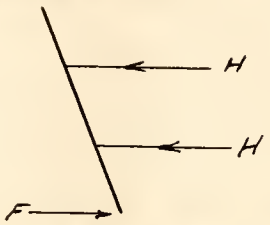
A quadrilateral box without a cover or a bottom is to be formed, having two opposite sides inclined, and the other two vertical, all of which will serve as retaining walls. The opposite sides will be connected with horizontal steel rods, and, having no other connection, will be able to move under earth pressures from within

the box in the direction of the rods by elogating the steel. The bars are to be spaced vertically so that the angle of inclination of the wall will remain constant. The box may be filled with any material as sand, earth, gravel or broken stone. The deformation of the steel rods caused by the filling will be a measure of the horizontal component of the resultant pressure, and will be registered by extensometers attached to the rods; the vertical component is to be measured by scales placed under the sides of the box.

From the obtained horizontal and vertical components of the resultant, the direction and magnitude of the resultant itself can be easily computed.

The results then are to be compared with the results given by the variuos theories, and if they do not agree, deduce expressions which will furnish means of computing the actual pressure which a given retaining wall is to resist.

In the described arrangement the condition of forces on the back of the wall are as shown in Fig. 4.



The resultant pressure will be resolved into its vertical and horizonta components, which in turn will be divided into as many parts as the number of rods in a vertical section. Under the action of the pressure the two opposite sides of the box will move in opposite directions, causing an elongation of the steel. This movement will be hindered by the friction "F" between the edge of the wall and the floor; introducing rollers at this line of contact, force "F" becomes so small that it may be neglected, and the wall may be considered in a condition to move freely with the deformation of the steel rods.

The extensometers are to be of a type as manufactured by F.F.Metzger and Co. It is a small apparatus which registers the deformations on a dial to .0002 inch. The extensometers are to be enclosed in a wooden box, which is to be illuminated on the inside; in one of the sides of this box a metal funnel will be inserted from which a 2 inch gas pipe will lead thru the wall enabling the observer with the aid of a telescope to read the dial.

It is necessary for this experiment to determine the exact value of the modulus of elasticity and the elastic limit of the steel used. The first is to be used to compute the pressure from the deformations; the second is for the purpose of assigning a unit stress to the rods, as it is desirable to use the steel as a perfectly elastic material.

Section 2.

...

Computation of Pressures on the Inclined Wall.

The height of the box will be 12 feet, which is about the average height of ordinary retaining walls. All computations will be made for a sand fill weighing 100 lbs. per cubic foot, and having an angle of repose of 30°. Rankine's ellipse of stress will be made use of in the computations; the inclined wall will make an angle of 15° with the vertical. The ratio of the vertical to the horizontal force at any depth within a granular mass is

$$\frac{q}{p} = \frac{1 - \sin \theta}{1 - \sin \theta}$$

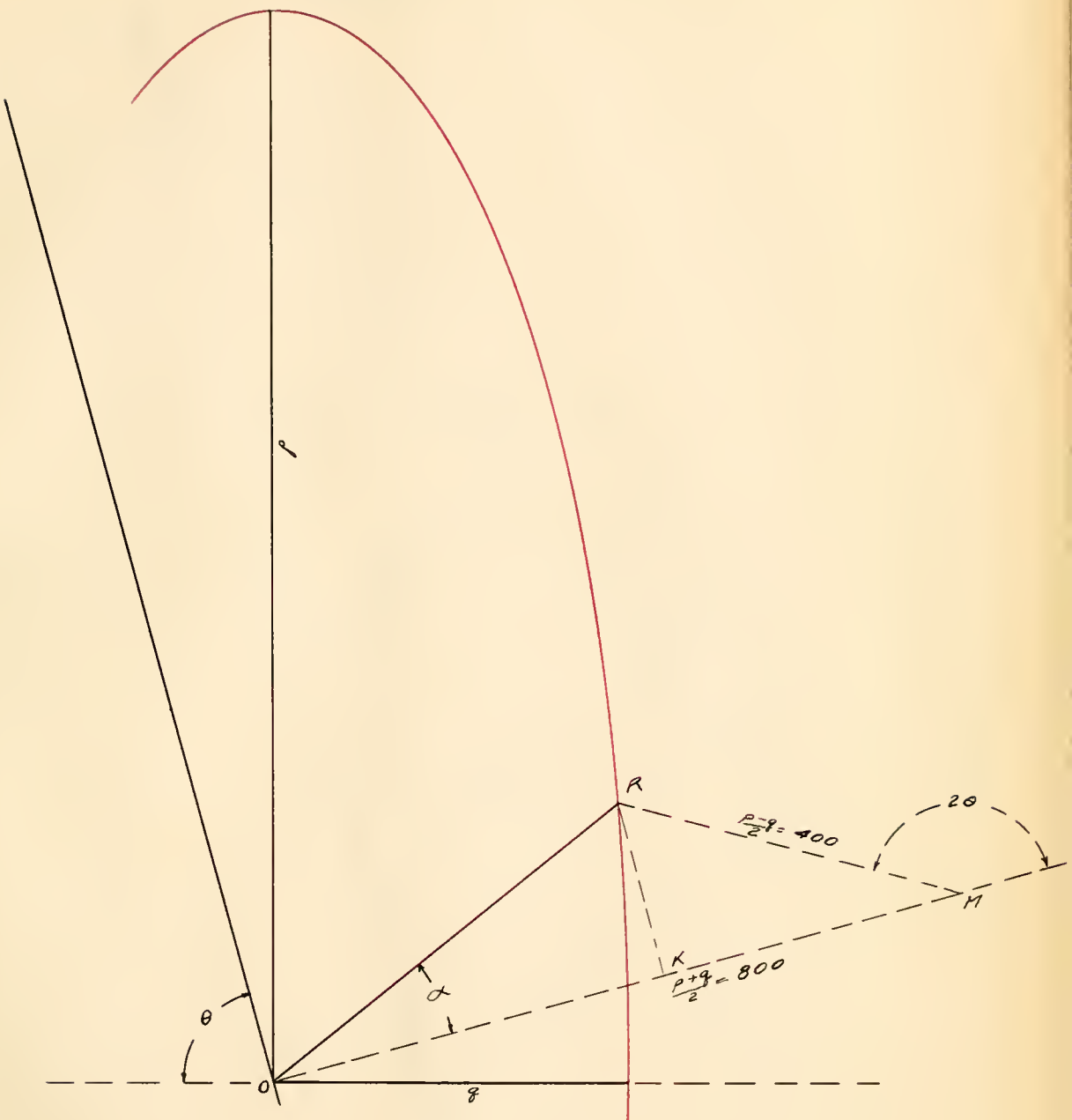
p equals 12 x 100 θ equals 30°

Therefore,

$$\frac{q}{1200} = \frac{.5}{1.5} \quad \text{and} \quad q = 400 \text{ lbs.}$$

Construct the ellipse of stress at a point 12 feet below the top of the wall. Fig. 5.

Fig. 5



$\rho = 1200''$

$q = 800''$

$KM = RM \times \cos 30° = 346.5''$

$RK = KM \times \tan 30° = 199.5''$

$OK = 800 - 346.5 = 453.5''$

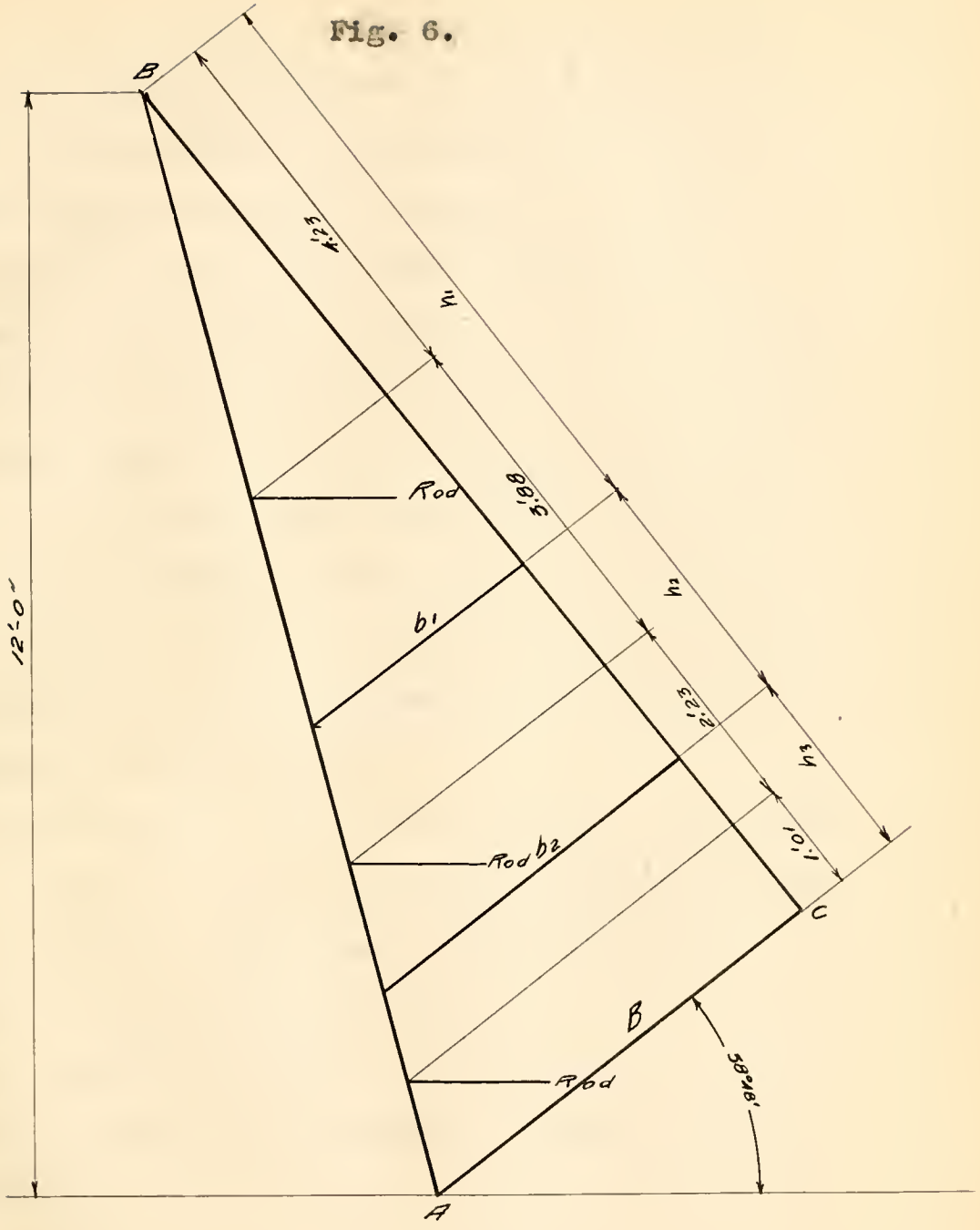
$\theta = 75°$

$\angle RMK = 30°$

$\alpha^{-1} = \frac{199.5}{453.5} = .44 \quad \therefore \alpha = 23°48'$

$OR = \frac{453.5}{\cos\alpha} = 496''$

Fig. 6.



If A C in Fig. 6. represents the pressure at the bottom, then the area of the triangle A B C is the total pressure on the back of the wall. Every rod placed in this triangle of pressure will take a part of the total horizontal component of this force. In order to keep the angle θ constant thruout the experiment, it is necessary to make these parts of the total pressure taken by each rod equal, which amounts to dividing the area of the triangle into three equal areas, as this number of rods will be used along a vertical section, and placing each rod at the center of gravity of these figures.

Let area B a b = a b c d = c d A C = B A C/3

h, h_2 and h_3 = the height of the respective areas.

b_1, b_2 and B = their bases expressed in lbs.

Then, H = h_1 plus h_2 plus h_3 =

(12 ÷ sin 75^0) sin $66^0 2$ = 11.35 feet.

From the geometrical relation of these areas we may write,

$$\frac{H\ B}{2 \times 3} = \frac{h_1\ b_1}{2} \qquad 1.$$

$$h_1 b_1 = \frac{H - h_1}{2} (b_1 + B)$$

2.

$$\frac{(b_2 - B)}{2} h_3 = \frac{H - h_3}{2} \frac{b_2}{2}$$

$$(b_2 - B) \frac{h_3}{2} = \frac{H B}{2 \times 3}$$

Solving the two pairs of equations, we get the heights of the equivalent areas and the pressures at the corresponding depths as follows:

$$h_1 = \frac{H}{\sqrt{3}} \qquad b_1 = \sqrt{\frac{1}{3}}\ B$$

$$h_2 = H - h_1 - h_3 \qquad b_2 = \sqrt{\frac{2}{3}}\ B$$

$$h_3 = \frac{H}{3\left(\sqrt{\frac{2}{3}} + 1\right)}$$

Substituting the value of H and B and transposing, we get the following:

$h_1 =$ 6.54 ft. $\qquad b_1 =$ 206 lbs.

$h_2 =$ 2.73 ft. $\qquad b_2 =$ 405 lbs.

$h_3 =$ 2.08 ft. $\qquad B =$ 496 lbs.

The area of each of these figures is 935 ,which is the total pressure on one foot of width of the wall to be taken by each rod.

The centers of gravity with reference to the triangle of pressure are as shown on Fig. 6. (4'.23 ; 3'.88 ; 2'.23 ; 1'.01)

Section 3.

.....

Computations of Pressures on the Vertical Wall.

The horizontal pressure at the bottom of the vertical side of the wall is in the ratio to the vertical pressure,

$$\frac{q}{p} = \frac{1 - \sin\theta}{1 + \sin\theta} = \frac{1 - .5}{1 + .5} \qquad q = \frac{1}{3} p$$

p = 12 x 100 for 1 foot of width.

Therefore, q = 1200 ÷ 3 = 400 lbs.

Since the back of the wall is vertical the resultant pressure is horizontal according to Rankine's Theory. Now if A B represents the back of the wall, and A C the pressure at the bottom, the area of the triangle represents the total pressure on the wall, which is to be divided into three equivalent areas at the centers of gravity of which the rods are to be placed. *Fig. 7*

By using the formulas obtained for the inclined wall, we get the dimensions of these equivalent areas as follows:

h_1 = 6.93 ft. b_1 = 231 lbs.

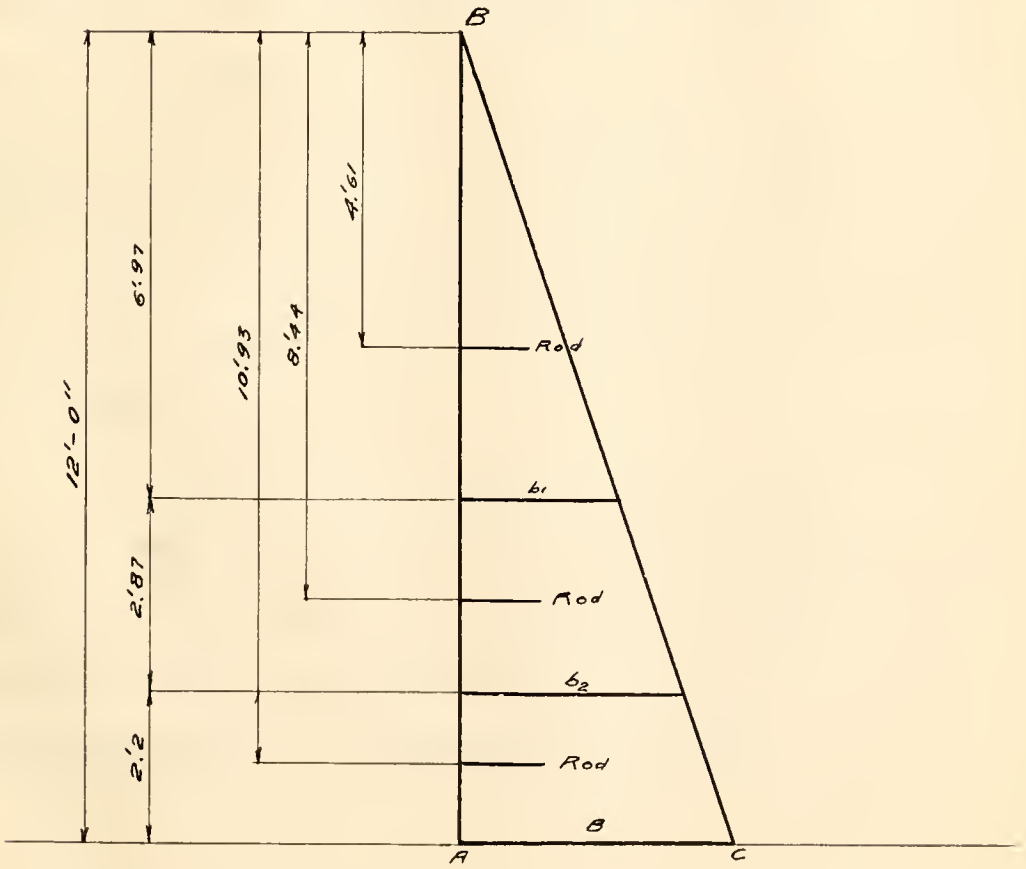
h_2 = 2.87 ft. b_2 = 327 lbs.

h_3 = 2.20 ft. B = 400 lbs.

The area of each is 800

The steel is to be placed at distances of 4.61 ft.; 8.44 ft.; and 10.93 ft. from the top of the wall.

Fig. 7.



Section 4.

....

Design of the Wall.

The walls will be constructed of boards the length of which will constitute the width of the box. At each vertical row of rods a plank will be placed, and the boards bolted to it.

In order to secure the full pressure of the fill on the wall, the length of it must be 2 x h x cotan θ , or, 2 x 12 x 1.732 41.6 ft. For convenience of spacing of the rods, the length of the wall will be 44 ft.

Three rows of rods, eleven in each row, will be used, thus giving to each rod four ft. of width of the wall.

The pressure per square foot for each bar as computed in Section 2 for the inclined wall will be 935 lbs. The bottom board will have to resist the greatest pressure and this board will be designed accordingly.

This board has a span of 4 feet and is 1 foot wide; the pressure at the lower edge of the board is 495 lbs. per square foot, and 455 lbs. at the top, which gives an average pressure of 475 lbs. per square foot. The normal component of the 475 lbs. produces bending of the board which is 475 x .915 = 430 lbs.

Ketchum's Retaining Walls and Bins (page 379), gives two formulas for the bending moments for this condition, namely,

$$M_c = \frac{w l^2}{24} \qquad M_s = -\frac{w l^2}{12}$$

where M_c is the bending moment at the center, and M_s is the bending moment over the supports.

The second formula is to be used, namely,

$$M = \frac{430 \times 16 \times 12}{12} = 6880'' \text{lbs.}$$

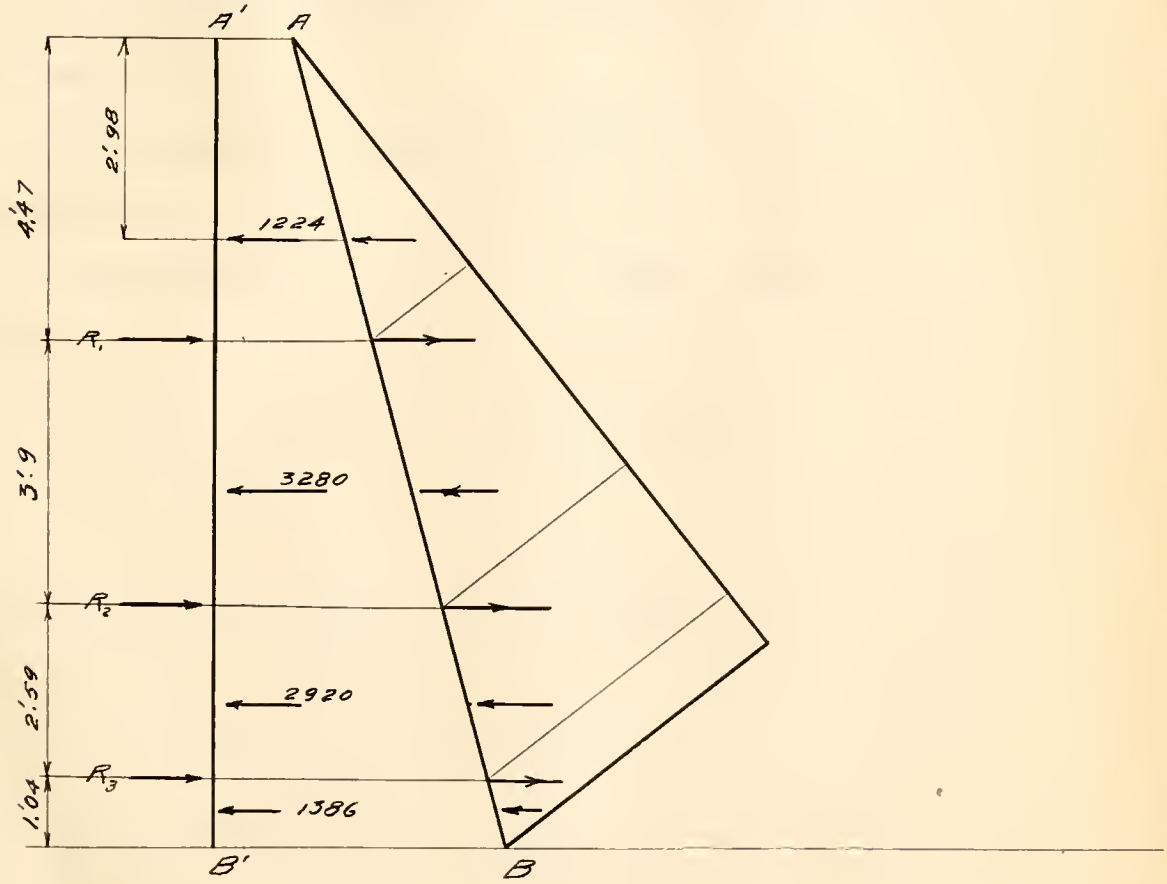
Using a board 12" x 1 $\frac{1}{2}$ "

$$I = \frac{b \times d^3}{12} = \frac{12 \times 27}{12 \times 8} = 3.38''^4$$

$$s = \frac{M\ c}{I} = \frac{6880 \times 3}{4 \times 3.88} = 1330 \text{ lbs.}$$

The ultimate strength of Douglas Fir is 6900 lbs. per square inch, giving a factor of safety of over 5.

Fig. 8.



The rods, as far as the inclined posts are concerned, will act as reaction, and the members as continuous beam. The vertical projection of the member A B will be considered in computing the bending. The bending moment produced at the first rod from the top is due to the horizontal component of the pressure A a b acting at the center of gravity of this triangle. In a similar manner the rest of the forces are to be considered.

The computed values of these forces are shown in Fig. 8.

The bending moment at the first support equals,

$$M = 1224 \times 1.49 \times 12 = 22,100'' \text{ lbs.}$$

$$s = \frac{M\,c}{I}$$

using for this member a 3" x 6" plank

$$I = \frac{3 \times 6^3}{12} = 54 \text{ inches}$$

$$s = \frac{22,100 \times 3}{54} = 1330 \text{ lbs.}$$

The same lumber will be used for the vertical wall.

Each rod is to be subjected to a tension of 7480 lbs. Using mild steel which has for its elastic limit a value of 30,000 lbs. per sq. in., and allowing a stress of 20,000 lbs. per sq. in., the cross-section of the rod will have to be .37 sq. in. A 11/16 in. rod has a cross-section of .376 sq. in., and is suitable for this use.

The rods for the vertical wall are to be subjected to a tension of 6400 lbs. each, and an area of .32 sq. in. is required. A 5/8 in. rod has a section of .31 sq. in., and hence may be used.

Note.

Figures 9 and 10 show vertical sections of the inclined and vertical sides of the box, and Fig. 11 shows a top view of it with the steel in place.

Section 5.

....

Computation of Pressures from Observed Data.

Let E be the modulus of elasticity of the steel.

Let a be the cross-section of the rod in square inches.

Let l be the average observed elongation of the rod.

Let n be the length of the extensometer. Then,

l/2 is the elongation caused by the pressure on one side, and l/2 a n the unit deformation of the rod.

From the theory of elastic change in length, we conclude that

$$E = \frac{s \times 2 \times n \times a}{l}$$

where s is the tension in the rod.

Since there were three rods along a vertical section for each 4 feet of width,

$$s = \frac{3\,E\,I}{8\,n\,a}$$

which is the horizontal component of the resultant pressure on the back of the wall for one foot of width.

Let this quantity be " H ", and the average vertical component of the resultant as measured by the scales be " V "

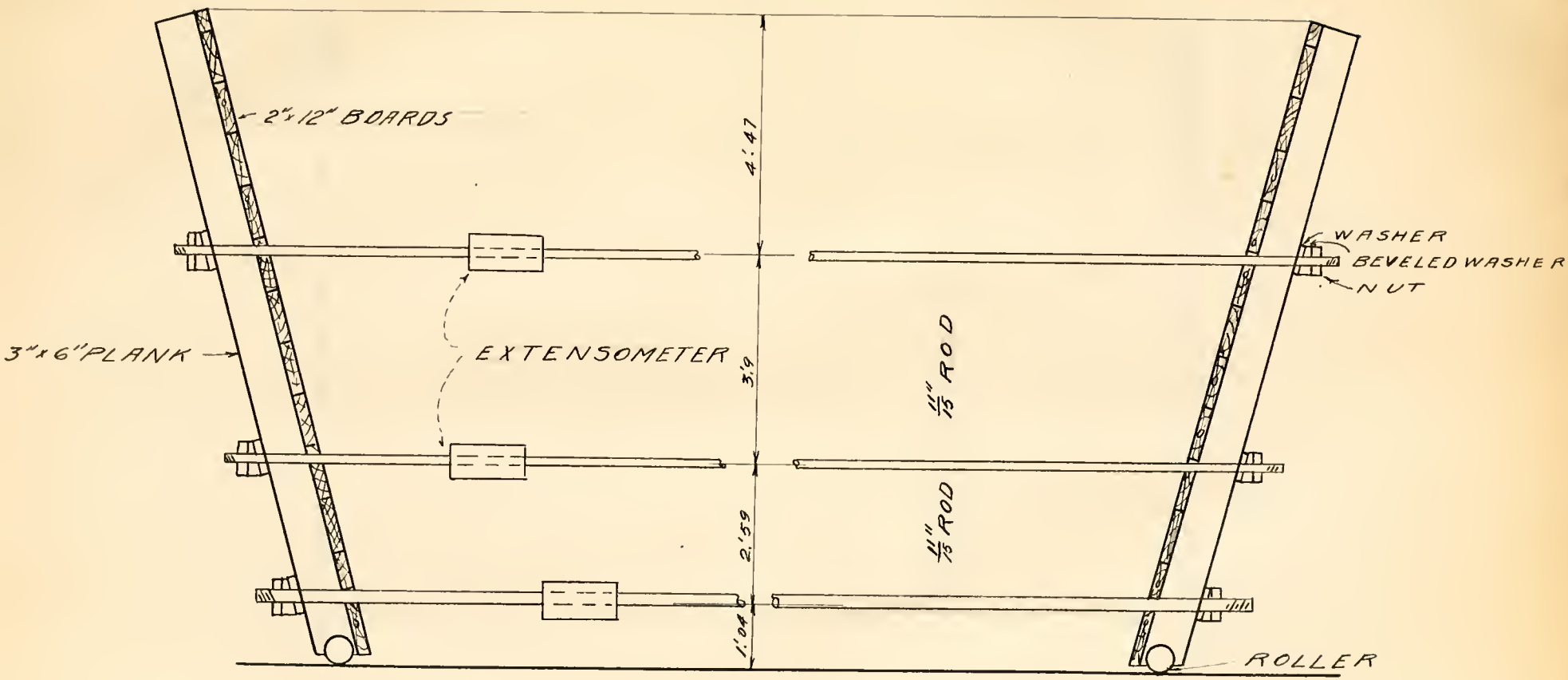
Then pressure R is

$$R = \sqrt{H^2 + V^2}$$

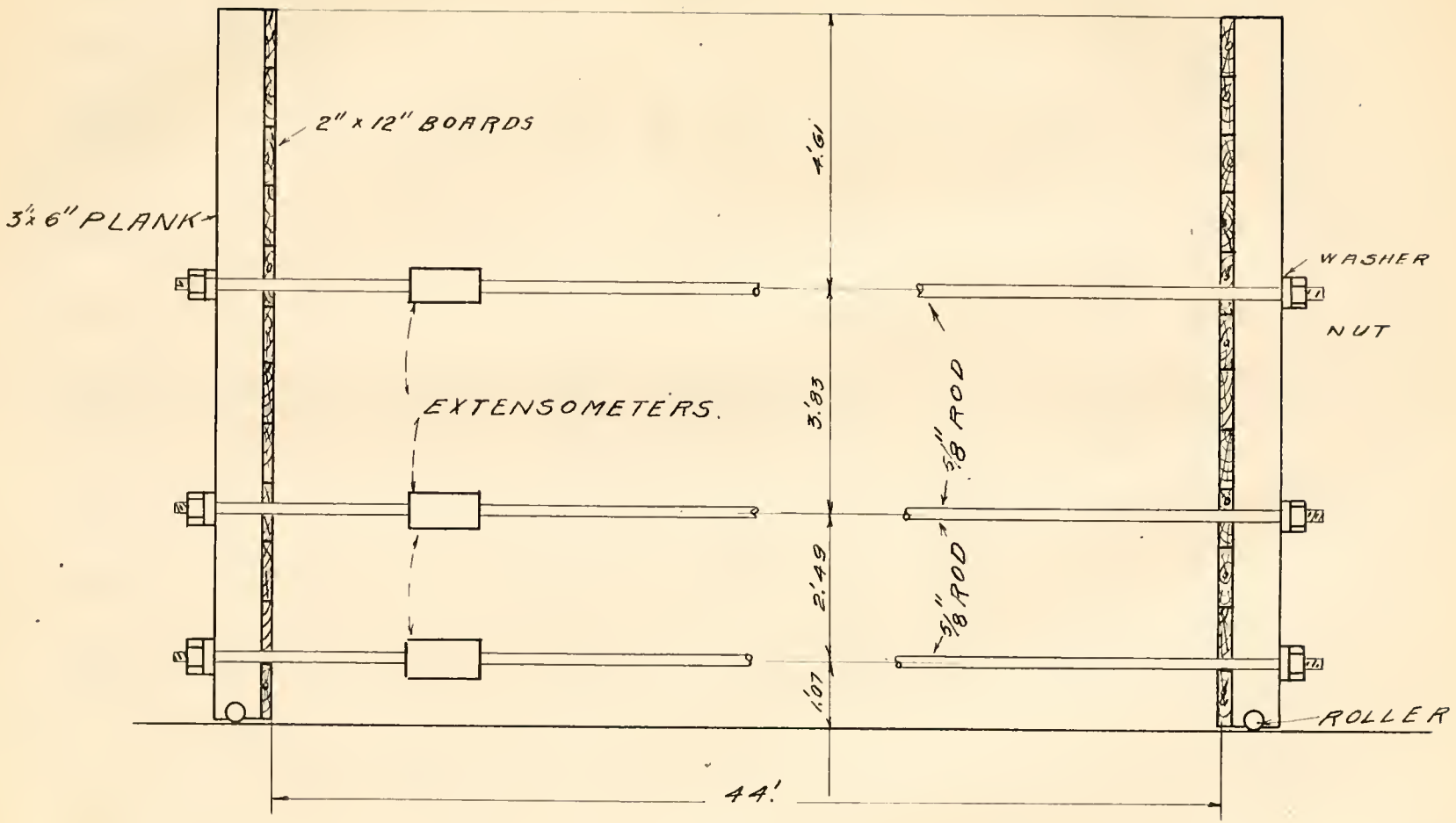
The angle which this resultant makes with the horizontal is given by

$$\tan^{-1} = \frac{V}{H}$$

Fig. 9.



VERTICAL SECTION OF INCL. SIDES OF BOX



VERTICAL SECTION OF VERTICAL SIDES OF BOX

Fig. 10

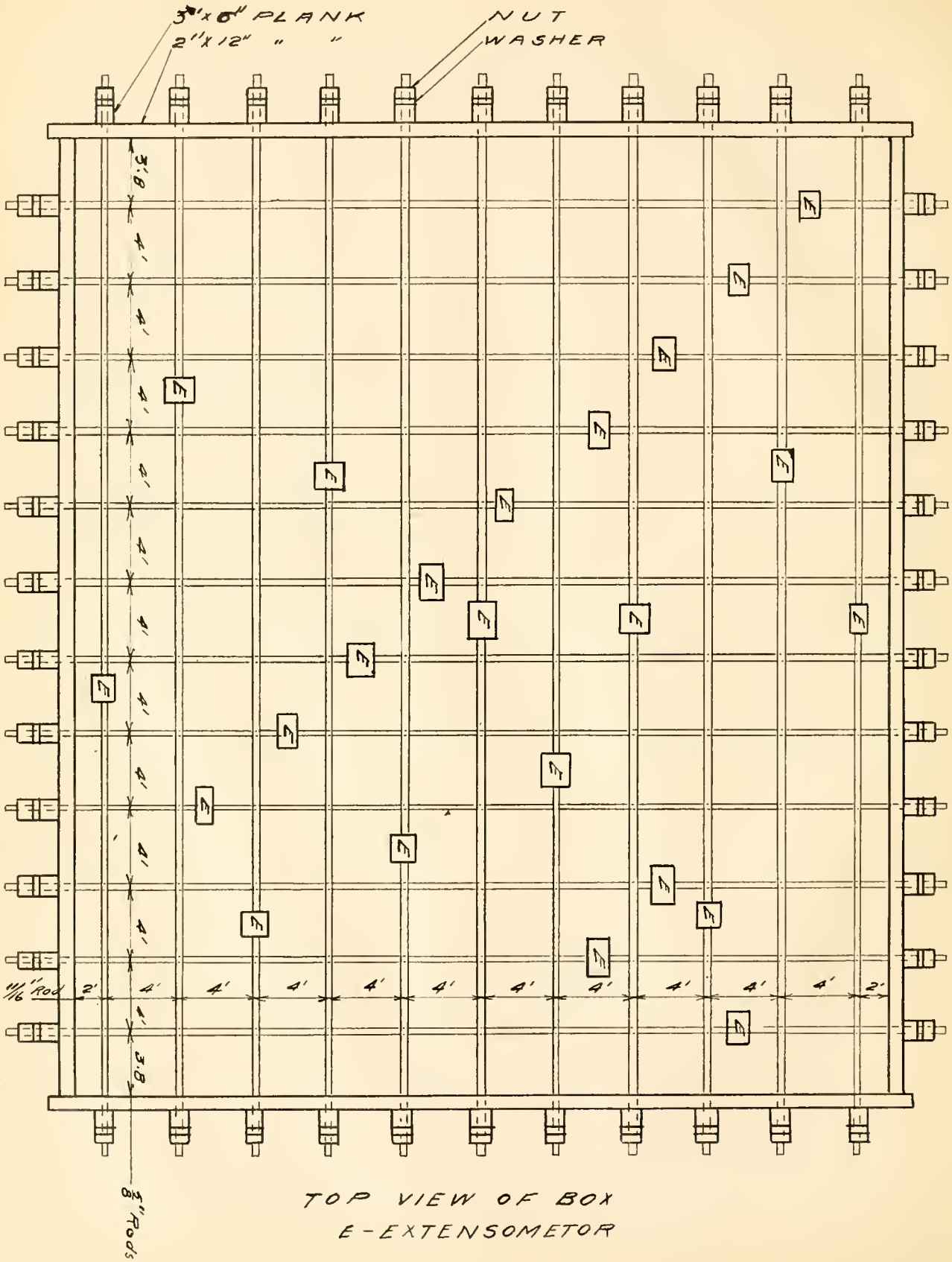


Fig 11

TOP VIEW OF BOX
E-EXTENSOMETOR